

# Rapidity distribution of gluons in the classical field model for heavy ion collisions

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The rapidity distribution of gluons produced in heavy ion collisions is studied by a numerical computation in 2+1-dimensional classical Yang-Mills theory. By assuming that the classical source strength  $g^2\mu$  depends on rapidity as  $g^4\mu^2 \sim e^{\pm\lambda y}$  and studying collisions of two nuclei with different  $g^2\mu$  we find that the rapidity distribution of produced gluons at central rapidities is very broad. The transverse energy is seen to decrease even more slowly as a function of  $y$  than the multiplicity. We discuss these results and the range in  $y$  and  $\sqrt{s}$  where they are applicable in the light of experimental results and other theoretical calculations.

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## I. INTRODUCTION

The longitudinal distribution of particles produced in a relativistic heavy ion collision is experimentally most easily measured as a function of the pseudorapidity  $\eta$ , because measuring pseudorapidity does not require particle identification. This measurement has been done for a wide range in  $\eta$  e.g. by the PHOBOS experiment [1]. Theoretical calculations, on the other hand, naturally produce rapidity distributions. Given the different particle species and the different stages of the collision from the initial conditions to hadronisation and decoupling, finding the right way to transform between pseudorapidity and rapidity is not always simple. The uncertainty in interpreting experimental pseudorapidity distributions in terms of theoretical calculations of rapidity dependence is discussed in e.g. [2]. More recently, however, the BRAHMS [3] and STAR [4] experiments have measured charged particle yields also as a function of the rapidity  $y$ . Especially the BRAHMS data, covering a wide range in  $y$ , facilitates the comparison between theoretical models and experimental results.

The aim of this paper is to calculate numerically the rapidity dependence of gluon production in the classical field model around central rapidities. In Sec. II we briefly review this model and how it can be applied to study gluon production in heavy ion collisions. In Sec. III we discuss the relation between the McLerran-Venugopalan model and saturation and argue that the rapidity dependence of particle production can be studied by varying the strengths of the classical sources as  $g^4\mu^2 \sim e^{\pm\lambda y}$ , at least for large enough  $\sqrt{s}$  and for small enough rapidity. Numerical results from applying the classical field model to collisions of two nuclei with different color charge densities are presented in Sec. IV and these results and their applicability to heavy ion phenomenology discussed in Sec. V.

## II. THE CLASSICAL FIELD MODEL

The McLerran-Venugopalan model for the small  $x$  wavefunction of an ultrarelativistic nucleus was suggested in [5, 6, 7]. The classical field model for the initial stage of a collision of two heavy ions, based on the McLerran-Venugopalan model for the nuclear wavefunction, was formulated in [8] and in [9, 10].

Let us assume we have two nuclei moving along the light cone, corresponding to a current

$$J^\mu = \delta^{\mu+} \delta(x^-) \rho_{(1)}(\mathbf{x}_T) + \delta^{\mu-} \delta(x^+) \rho_{(2)}(\mathbf{x}_T). \quad (1)$$

The two colour charge densities  $\rho_{(m)}(\mathbf{x}_T)$  are, independently for the two nuclei, drawn from a random ensemble, which in the original McLerran-Venugopalan model is taken to be Gaussian:

$$\langle \rho_{(m)}^a(\mathbf{x}_T) \rho_{(m)}^b(\mathbf{y}_T) \rangle = g^2 \mu_{(m)}^2 \delta^{ab} \delta^2(\mathbf{x}_T - \mathbf{y}_T), \quad m = 1, 2, \quad (2)$$

where  $\mu$  is a parameter describing the transverse density of color charges<sup>1</sup>.

We then want to find the color fields generated by this current using the classical equations of motion

$$[D_\mu, F^{\mu\nu}] = J^\nu. \quad (3)$$

In the light cone gauge ( $A^+ = 0$  for nucleus (1),  $A^- = 0$  for nucleus (2)) one first calculates the pure gauge fields corresponding to the two nuclei:

$$A_{(m)}^i(\mathbf{x}_T) = \frac{i}{g} U_{(m)}(\mathbf{x}_T) \partial_i U_{(m)}^\dagger(\mathbf{x}_T), \quad m = 1, 2. \quad (4)$$

These depend on the Wilson lines in the covariant gauge,

$$U_{(1)}(\mathbf{x}_T) = P \exp \left\{ i \int dx^- A_{\text{cov}}^+(\mathbf{x}_T) \right\} \quad (5)$$

$$U_{(2)}(\mathbf{x}_T) = P \exp \left\{ i \int dx^+ A_{\text{cov}}^-(\mathbf{x}_T) \right\},$$

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<sup>1</sup> The relation to the convention introduced in [11] is  $g^2\mu = \Lambda_s$ .

which can, for infinitely Lorentz-contracted nuclei, be calculated as

$$U_{(m)}(\mathbf{x}_T) = \exp \left\{ -ig \frac{\rho_{(m)}}{\nabla_T^2}(\mathbf{x}_T) \right\}. \quad (6)$$

In a temporal gauge  $A_\tau = 0$  the initial condition at  $\tau = 0$  for the color fields  $\mathbf{A}_T(\tau, \mathbf{x}_T)$  and  $A_\eta(\tau, \mathbf{x}_T)$  is given by these pure gauge fields corresponding to the two nuclei:

$$\begin{aligned} A^i(0, \mathbf{x}_T) &= A_{(1)}^i(\mathbf{x}_T) + A_{(2)}^i(\mathbf{x}_T), \\ A^\eta(0, \mathbf{x}_T) &= \frac{ig}{2} [A_{(1)}^i(\mathbf{x}_T), A_{(2)}^i(\mathbf{x}_T)]. \end{aligned} \quad (7)$$

To find the gauge field in the future light cone  $\tau > 0$  one then solves the gauge field equations of motion using these initial conditions. In the gauge  $A_\tau = 0$  it is easy to find the Hamiltonian and thus the energy of a given field configuration. Additionally, fixing the Coulomb gauge in the transverse plane,  $\nabla_T \cdot \mathbf{A}_T = 0$ , one can also define a gluon multiplicity corresponding to the classical fields. The method of solving the classical Yang-Mills equations numerically has been developed in [12]. First numerical results for SU(2) were found in [13, 14] and for SU(3) in [11, 15, 16].<sup>2</sup> The numerical code and notations in this paper are those used in [15].

### III. SATURATION AND RAPIDITY DEPENDENCE

The saturation scale  $Q_s$  is an important concept in small  $x$  physics (see e.g. [18, 19] for a review). It has been found to depend on the value of  $x$  probed in the process as  $Q_s^2 \sim x^{-\lambda}$  both in analytical studies of the quantum evolution of the color sources in the classical field model [20, 21, 22, 23, 24, 25, 26]<sup>3</sup> and in phenomenological studies of DIS data [27, 28]. The value of  $\lambda$  found in [27] is  $\lambda = 0.277 \dots 0.288$ . In [29, 30] the values used are  $\lambda = 0.25 \dots 0.3$ . In presenting the numerical results in Sec. IV we shall keep  $\lambda$  arbitrary and use it only in Sec. V when comparing with experimental results and other theoretical calculations.

In the McLerran-Venugopalan model the saturation scale  $Q_s$  is related to the strength of the classical sources by

$$Q_s^2 = \frac{g^4 \mu^2 C_A}{4\pi} \ln \left( \frac{g^4 \mu^2}{\Lambda_{\text{QCD}}^2} \right). \quad (8)$$

Note that this is the saturation scale as defined in e.g. [10], and differs from the one used in the context of DIS (e.g. [26]) by the color factor  $C_A$  vs.  $C_F$ <sup>4</sup>. The relation (8) can be derived by calculating the correlator of

the Wilson lines, Eq. (5), in the adjoint representation (see [32] for the detailed calculation). Up to a logarithmic uncertainty which we will neglect in this study, the strengths of the classical sources,  $g^2 \mu_{(1,2)}$ , should thus also depend on the  $x$  probed in nuclei (1) and (2) respectively. In the perturbative weak field limit of the model we are considering here the partons are produced in a  $2 \rightarrow 1$  process, with a gluon produced at a given  $p_T$  and  $y$  coming from gluons in the initial nuclear wave functions at [33]:

$$x_{1,2} = p_T e^{\pm y} / \sqrt{s}. \quad (9)$$

Assuming that the dominant transverse momentum scale  $\langle p_T \rangle$  does not depend on  $y$  one is lead to assume that to calculate gluon production at a rapidity  $y$  one must take

$$\mu_{(1)}^2 = \mu^2 e^{\lambda y} \text{ and } \mu_{(2)}^2 = \mu^2 e^{-\lambda y}, \quad (10)$$

where  $\mu_{(1,2)}$  are the source strengths appearing in the correlator in Eq. (2). Note that the geometric mean  $\mu \equiv \sqrt{\mu_{(1)} \mu_{(2)}}$  is the quantity appearing in Eqs. (11), (12). Our assumption that  $\langle p_T \rangle$  does not depend on  $y$  is valid only at small  $y$  in AA-collisions where, by symmetry, one expects  $\langle p_T \rangle_y \sim (1 + \mathcal{O}(\lambda^2 y^2)) \langle p_T \rangle_{y=0}$ . Further away from central rapidities one could assume  $\langle p_T \rangle$  to depend on one of the saturation scales of the two nuclei, and the relation (10) cannot be used any more. A separate question from the selfconsistency of the calculation discussed above is whether  $\sqrt{s}$  at RHIC is high enough for this model to be applicable; we will return to this question in Sec. V.

### IV. RESULTS

We calculate the transverse energy and multiplicity per unit rapidity as in [15], and show our results in terms of the dimensionless ratios

$$f_E = \frac{1}{g^4 \mu^3 \pi R_A^2} \frac{dE}{d\eta} \quad (11)$$

and

$$f_N = \frac{1}{g^2 \mu^2 \pi R_A^2} \frac{dN}{d\eta}. \quad (12)$$

These depend on  $\lambda y$  through the source strength (by Eq. (10)) and the dimensionless parameter characterising the field strength  $g^4 \mu^2 \pi R_A^2$ . We shall consider  $g = 2$  and  $\pi R_A^2 = 140 \text{ fm}^2$  as constants and vary the source strength by using different values of  $\mu$ . As in [15], the numerical computation presented here is done assuming cubic nuclei, whose projection to the transverse plane fills the whole 2-dimensional lattice (of transverse area  $\pi R_A^2$ ), enabling the use of periodic boundary conditions.

Figure 1 shows the behaviour of  $f_N$  and Fig. 2 of  $f_E$  as a function of  $\lambda y$  for transverse lattices of  $256^2$  and  $512^2$

<sup>2</sup> See also the erratum [17].

<sup>3</sup> See e.g. [19] for a more comprehensive list of references.

<sup>4</sup> This difference is discussed in more detail in e.g. [19, 31].

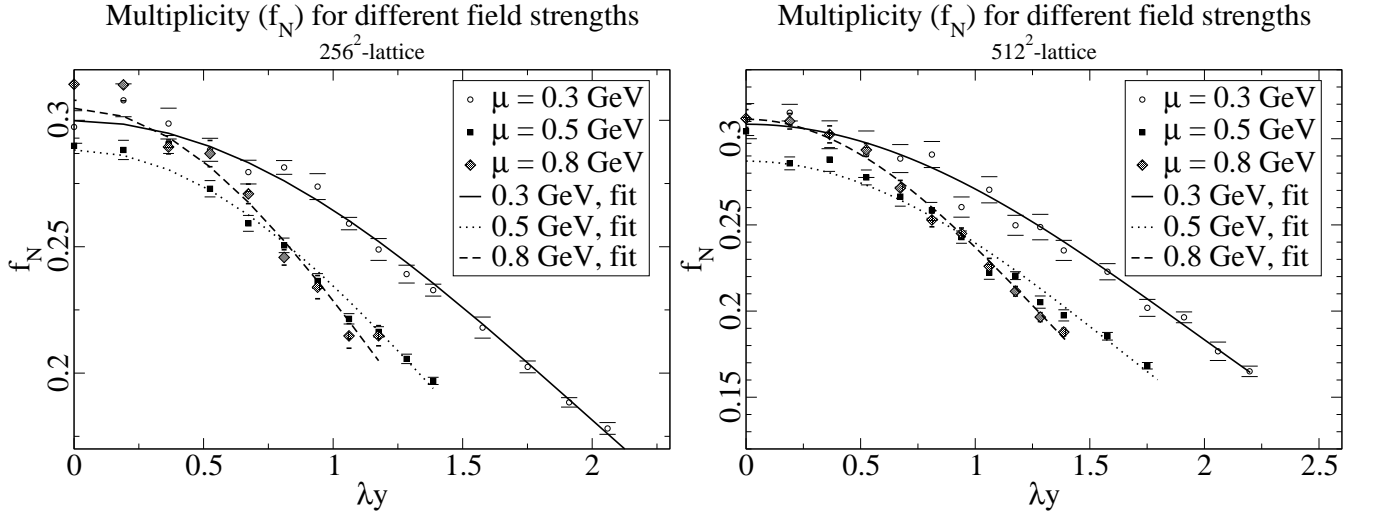


Figure 1: The dimensionless ratio  $f_N$  for different rapidities on two different lattice sizes (left:  $256^2$ , right:  $512^2$ ) and for different field strengths. Also shown are the Gaussian fits with the widths in Table I.

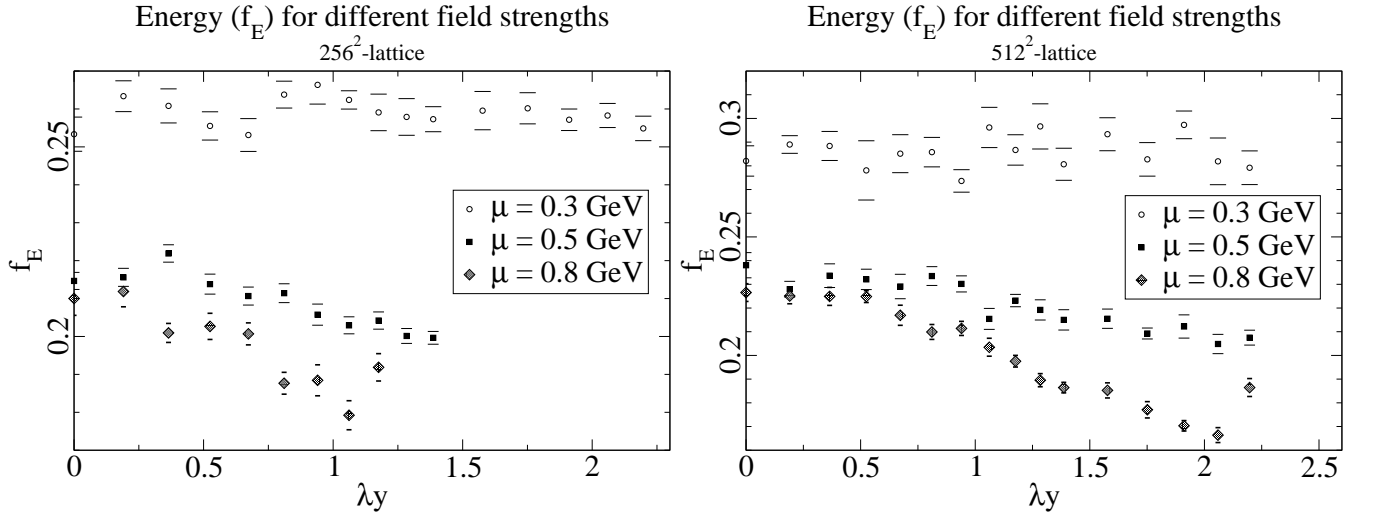


Figure 2: The dimensionless ratio  $f_E$  on two different lattice sizes (left:  $256^2$ , right:  $512^2$ ) and for different field strengths.

points. One can see that whereas the multiplicity decreases slowly with rapidity, the transverse energy does not change significantly, meaning that the energy per particle increases. This increase is so slow, however, that our approximation in Sec. III of considering  $\langle p_T \rangle$  roughly independent of  $y$  is justified for  $\lambda y \lesssim 1$ . The behavior of the gluon spectrum is illustrated in Fig. 3, which shows the  $\mathbf{k}_T$ -distribution of gluons for two different rapidities. This hardening of the spectrum is an inherent feature in a distribution depending on two transverse momentum scales  $g^2\mu_{(1,2)}$ . Note that the presence of two widely different momentum scales also poses difficulties for the numerical calculation, as the harder scale approaches the lattice ultraviolet cutoff and the softer scale approaches the infrared divergent weak field result. For this reason, although one can use this method to study gluon spectra and the Cronin effect in “pA”-collisions (with one

of the sources very weak), as has been done in [16], it is perhaps not realistic to calculate integrated multiplicities for the “pA”-case. Instead, saturation physics in “pA”-collisions<sup>5</sup> can be studied using a qualitatively different treatment for the proton and the nucleus (see e.g. [38, 39, 40, 41]).

We fit the rapidity dependence of the multiplicity with a Gaussian form

$$f_N = f_0 \exp\left(-\frac{y^2}{2\sigma^2}\right) = f_0 \exp\left(-\frac{(\lambda y)^2}{2(\lambda\sigma)^2}\right). \quad (13)$$

Because the result only depends on the combination  $\lambda y$ ,

<sup>5</sup> Actually the relevant RHIC experiments, see e.g. [34, 35, 36, 37], are d+Au-collisions.

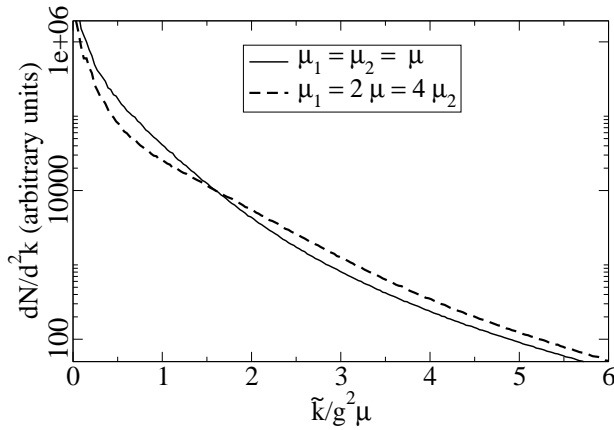


Figure 3: The spectra  $dN/d^2k_T$  of produced gluons for a symmetric collision ( $\mu_{(1)} = \mu_{(2)}$ ) and an asymmetric one ( $\mu_{(1)} < \mu_{(2)}$ ) plotted as a function of  $\tilde{k}/g^2\mu$ , where  $\tilde{k}^2 = 4 \sum_i \sin^2 k_i/2$ .

$\mu$	$g^4\mu^2\pi R_A^2$	$\lambda\sigma$ , 256 <sup>2</sup> -lattice	$\lambda\sigma$ , 512 <sup>2</sup> -lattice
0.3 GeV	5184	$1.99 \pm 0.02$	$1.96 \pm 0.03$
0.5 GeV	14400	$1.56 \pm 0.02$	$1.66 \pm 0.02$
0.8 GeV	36864	$1.32 \pm 0.03$	$1.35 \pm 0.02$

Table I: The widths of the Gaussians  $\lambda\sigma$  fitted to the multiplicities in Fig. 1.

with  $\lambda$  arbitrary so far, the result from the fit is actually the combination  $\lambda\sigma$ . The values of  $\lambda\sigma$  from the fits are listed in Table I. One sees that there is a small dependence lattice size and a surprisingly strong dependence on the field strength  $g^4\mu^2\pi R_A^2$ . The width  $\lambda\sigma$  seems to depend on the field strength more than  $f_E(y=0)$  and  $f_N(y=0)$ . This means that  $\lambda\sigma$  is more sensitive to the lattice infrared cutoff than the energy and multiplicity themselves. Note that at least in this context there is no fundamental reason for the Gaussian form, Eq. (13); by symmetry the distribution must be even in  $y$  and here we are mainly interested in the second derivative at  $y=0$ . Because of the numerical reasons discussed above we do not want to go to very large values of  $y$  and a form like  $[1 + y^2/(2n\sigma^2)]^{-n}$  for some  $n$  would be just as good. We choose the Gaussian because it has also been used in experimental [3] and other theoretical studies [42]. The rapidity distribution of the energy does not seem to naturally lend itself to a simple fit that would illustrate its structure any more than Fig. 2.

## V. COMPARISON WITH EXPERIMENTAL RESULTS AND OTHER THEORETICAL CALCULATIONS

It was found in [15] that assuming ideal hydrodynamical expansion and consequently entropy conservation RHIC multiplicities are best reproduced by taking  $g^2\mu \approx 2$  GeV i.e.  $\mu \approx 0.5$  GeV. Taking the corresponding value  $\lambda\sigma \approx 1.66$  from Table I and  $\lambda \approx 0.25 \dots 0.3$  one gets  $\sigma \approx 5.5 \dots 6.6$ .

The dependence of the saturation scale on rapidity has been exploited to study heavy ion phenomenology also by Kharzeev & Levin [29]. The authors use a saturation-inspired gluon distribution and perform a perturbative calculation of gluon production. The same approach has also been used by Hirano & Nara [30] to obtain initial conditions for a hydrodynamical calculation. Estimating from Fig. 1a of [30] the width of the distribution seems to be  $\sigma \approx 3$ , but the shape seems to be flatter than a Gaussian of this width at small  $y$  and fall more rapidly at large  $y$  than a Gaussian.

Our calculation differs from these papers in that we are performing a numerical calculation by solving the classical field equations, not a perturbative calculation using unintegrated gluon distributions. One advantage in our calculation is that the multiplicity comes out naturally as a smooth function in  $y$  around  $y=0$ , without the  $e^{-\lambda|y|}$ -discontinuity in the first derivative in [29]. In a perturbative calculation one can try to incorporate features of high- $x$  physics, such as the  $(1-x)^4$ -behaviour of the parton distributions imposed by hand in [29, 30], whereas our calculation stays within the framework of a small  $x$  model without including this kind of effects. This means that our calculation is limited to regions around  $y=0$ , where the fields of both nuclei are strong and can be treated classically, and such values of  $\sqrt{s}$  that large  $x$  effects are not important at midrapidity.

These results can also be compared to the pQCD+saturation model calculation [42] result of  $\sigma = 5.9$ . Whereas in [42]  $\sigma$  increases slightly when the saturation scale grows, in our calculation it decreases and the distribution becomes more peaked.

It is not straightforward to compare this calculated initial state gluon distribution to measured rapidity distributions in the final state. The approach of [29] is to assume that they are equal and only correct for the transformation between rapidity and pseudorapidity; in the hydrodynamical calculation of [43] the distribution broadens, but only slightly, during the hydrodynamical evolution. The result of the BRAHMS collaboration [3] for the rapidity distribution of charged pions is  $\sigma \approx 2.3$ , which is close to the result of [30] and considerably less than our result. This could be interpreted as an indication that the main features of the rapidity dependence of particle production in AA-collisions at RHIC are not dominated by saturation physics but by the large  $x$  behavior of the parton distribution functions that cause the multiplicity to fall faster for large rapidities than the

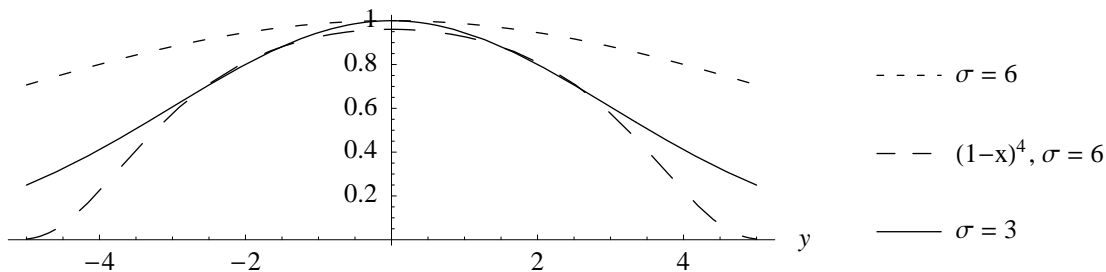


Figure 4: The first dashed curve labeled “ $\sigma = 6$ ” is the Gaussian  $e^{-y^2/(2 \cdot 6^2)}$ . The second curve, labeled “ $(1-x)^4$ ,  $\sigma = 6$ ” is the same Gaussian multiplied by  $(1 - e^{y/200})^4(1 - e^{-y/200})^4$ , where the factor  $1/200$  comes from assuming  $\sqrt{s}/A = 200$  GeV and  $\langle p_T \rangle \approx 1$  GeV. The solid curve is  $e^{-y^2/(2 \cdot 3^2)}$ , a Gaussian with width  $\sigma = 3$ . This figure is not intended as a reproduction of the calculation of [29, 30], but rather as a qualitative demonstration.

classical field model would suggest. The crucial importance of the factor  $(1-x)^4$  in the distribution functions of [29, 30] is illustrated in Fig. 4, which shows how, at RHIC energies, this factor can transform a broad Gaussian with  $\sigma = 6$  into a form that is closer to  $\sigma = 3$ . Note that the situation at the LHC will be quite different as the values of  $x$  probed at central rapidities will be an order of magnitude smaller than at RHIC and thus much less influenced by large  $x$  physics. Another way of viewing this is that energy conservation limits the size of a rapidity plateau at central rapidities and forces the multiplicity to decrease faster with increasing  $|y|$  [42].

In this calculation the average transverse momentum of the gluons,  $\langle p_T \rangle = \frac{f_E}{f_N} g^2 \mu$  increases for larger rapidities. The experimentally measured mean  $p_T$  in the final state is approximately constant or decreases. Although one can, in the hydrodynamic scenario, argue that a large part of the transverse energy goes into the longitudinal expansion of the system, it is hard to understand how the energy could decrease by a larger amount for larger rapidities. In the calculation of [30] the transverse energy decreases faster for larger  $y$  due to the explicit  $(1-x)^4$ -factor and momentum cutoffs in the transverse integration.

## VI. CONCLUSION AND OUTLOOK

We have presented a numerical calculation of the rapidity dependence of gluon production in heavy ion collisions. The calculation is performed in a 2+1-dimensional classical Yang-Mills model assuming that the strength of the classical sources vary with rapidity according to the simple relation  $g^2 \mu \sim e^{\pm \lambda y}$ . Our result is that the multi-

plicity is approximately a very broad Gaussian in rapidity with a width  $\sigma \approx 6$ . The distribution resulting from the classical field model is broader than the one observed experimentally. This could indicate that the rapidity distribution of particle production in heavy ion collisions at RHIC experiments depends more on the  $(1-x)^4$ -like large  $x$  behavior of parton distributions than actual saturation physics, i.e. the  $Q_s^2 \sim x^{-\lambda}$ -dependence of the saturation scale. At the LHC the situation could be quite different. Note, however, that our calculation only applies to central rapidities in AA-collisions and does not address pA-collisions.

The transverse energy produced in this model does not decrease for larger rapidities as fast as one would physically expect. This can be due to the model not including some relevant large  $x$  physics. Another potential reason could be that a 2+1-dimensional model is not sufficient to capture all the aspects of the longitudinal dependence of the problem, as is the case e.g. for quark pair production [44]. This will be understood better if one is able to perform a full 3+1-dimensional classical field computation with a more detailed inclusion of the quantum evolution of the sources. This calculation would also be important for understanding the subsequent thermalisation of the gluons produced in the collision.

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